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Structural Synthesis by Combining Approximation Concepts and Dual Methods

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Approximation concepts and dual methods are combined to create an efficient procedure for minimum weight sizing of structural systems. Approximation concepts convert the basic mathematical programming statement of the structural synthesis problem into a sequence of explicit primal problems of convex and separable form. These problems are solved by constructing explicit dual functions which are maximized subject to non-negativity constraints on the dual variables. A specially devised Newton type maximization algorithm called DUAL 2 operates in a sequence of dual subspaces, with gradually increasing dimensionality, such that the maximum dimensionality of the dual subspace never exceeds the number of strictly critical constraints by more than one. The power of the method presented is demonstrated by giving numerical results for several example problems, including a metallic swept wing and a thin delta wing with fiber composite skins.

Introduction

It is now widely recognized that a significant class of structural design optimization problems can be properly posed as nonlinear mathematical programming problems. These problems usually involve minimization, with respect to many design variables, of an explicit objective function, such as total weight, subject to a large number of inequality constraints that guard against various failure modes under each of several distinct loading conditions.

In recent years the introduction of approximation concepts (see, for example, Refs. 1-4) has led to the development of efficient methods of structural synthesis without loss of the generality inherent to the mathematical programming formulation. These approximation concepts have included: 1) reduction of the number of independent design variables by design variable linking; 2) temporary reduction of the number of inequality constraints by deletion techniques (e.g., regionalization and truncation, see Ref. 2); and 3) reduction of the number of finite element analyses by construction of high quality explicit approximations, for the inequality constraints retained, based on Taylor series expansions in terms of carefully selected intermediate variables (e.g., linked reciprocal design variables). Through the coordinated use of these approximation concepts the original problem statement [nonlinear program (NLP)] is replaced by a sequence of relatively small, explicit, approximate problems that retain the essential characteristics of the original design optimization problem. These approximate problems often have a special algebraic form (e.g., convex, separable, etc.).

Previously published work on the approximation concepts approach to structural synthesis has not fully exploited the special algebraic structure of the sequence of approximate problems generated. Herein the approximation concepts approach is combined with the recently reported dual formulation[5,6] to produce a very efficient method that takes full advantage of the special algebraic structure of each approximate primal problem. The joining together of the approximation concepts approach to structural synthesis and the dual formulation as implemented by the ACCESS 3 computer program for continuous variable problems is outlined in Fig. 1.

Formulation

It is well known[1-4] that a significant class of structural optimization problems can be accurately approximated by a primal nonlinear mathematical programming problem of the following special form:

Find $\boldsymbol{\alpha}$ such that

$$W(\boldsymbol{\alpha}) = \sum_{b=1}^{B} \frac{w_b}{\alpha_b} \rightarrow \min \tag{1}$$

subject to linear constraints

$$\bar{h}_q(\boldsymbol{\alpha}) = \bar{u}_q - u_q(\boldsymbol{\alpha}) \geq 0; \quad q \in Q_R \tag{2}$$

where

$$u_q(\boldsymbol{\alpha}) = \sum_{b=1}^{B} C_{bq}\alpha_b; \quad q \in Q_R \tag{3}$$

and the side contraints are written separately

$$\alpha_b^{(L)} \leq \alpha_b \leq \alpha_b^{(U)} \tag{4}$$

The w_b in Eq. (1) are positive fixed constants corresponding to the weight of the elements linked to the bth design variable when it equals unity ($\alpha_b = 1$). Equations (2) and (3) represent the current linearized approximation of the retained behavior constraints in which the C_{bq} are constants. The $\alpha_b^{(L)}$ and the $\alpha_b^{(U)}$, respectively, denote lower and upper limits on the independent reciprocal design variables (α_b), and Q_R denotes the set of retained behavioral constraints for the current stage. It should be kept in mind that Eqs. (1-4) represent the approximate primal problem for the pth stage of the overall iterative design process outlined in Fig. 1. Note that for statically determinate structures subject to static stress and displacement constraints the primal formulation given by Eqs. (1-4) is exact.

The foregoing approximate primal problem [i.e., Eqs. (1-4)] is strictly convex since the w_b are positive and the inequality constraints are linear. Since the approximate primal problem of Eqs. (1-4) is of separable form, the

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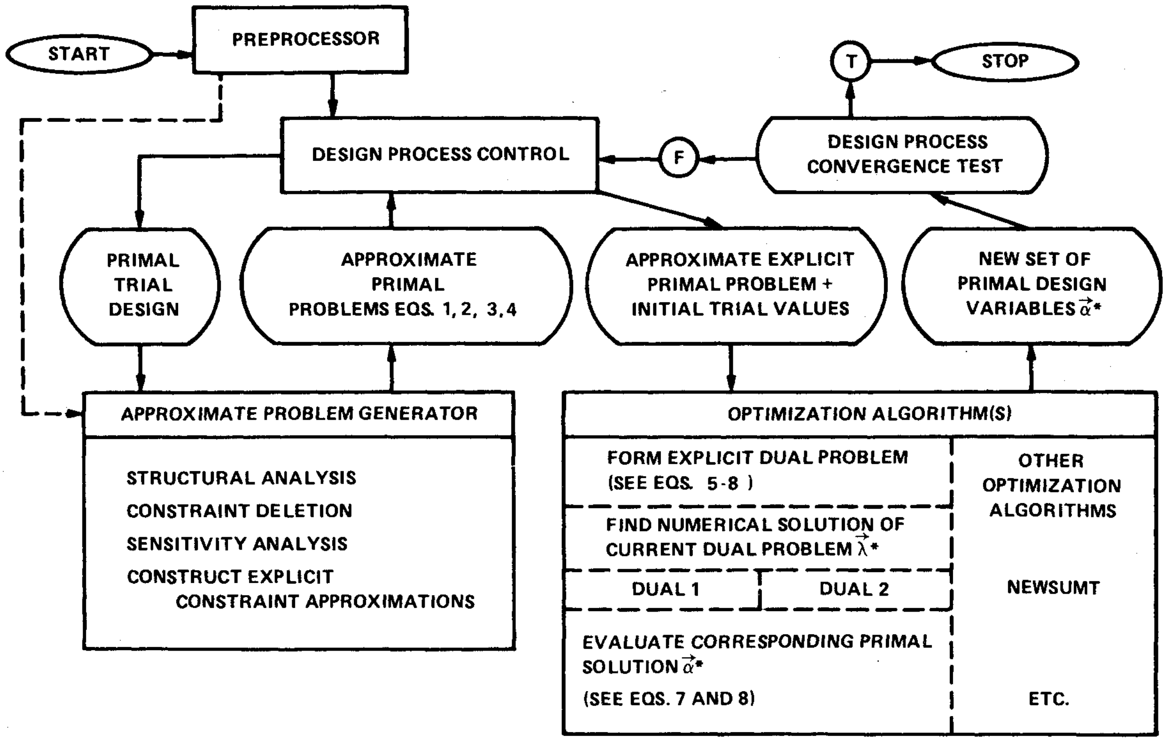

Fig. 1 Basic organization of ACCESS 3.

corresponding dual problem can be shown (see Sec. 2.4.2 of Ref. 7) to have the following form:

Find λ such that the explicit dual function

$$\ell(\lambda) = \sum_{b=1}^{B} \frac{w_b}{\alpha_b} + \sum_{q\in Q_R} \lambda_q [u_q(\alpha) - \bar{u}_q] \to \max \tag{5}$$

subject to nonnegativity constraints

$$\lambda_q \geq 0; \quad q \in Q_R \tag{6}$$

where $u_q(\alpha)$ is given by Eq. (3) and the primal variables (α_b) in terms of the dual variables (λ_q) are given by

$$\alpha_b = \begin{cases} \alpha_b^{(L)} & \text{if } [\alpha_b^{(L)}]^2 \geq \tilde{\alpha}_b^2 \\ \tilde{\alpha}_b & \text{if } [\alpha_b^{(L)}]^2 < \tilde{\alpha}_b^2 < [\alpha_b^{(U)}]^2 \\ \alpha_b^{(U)} & \text{if } \tilde{\alpha}_b^2 \geq [\alpha_b^{(U)}]^2 \end{cases} \tag{7}$$

where

$$\tilde{\alpha}_b^2 = w_b \Big/ \sum_{q\in Q_R} \lambda_q C_{bq} \tag{8}$$

The explicit dual function defined by Eqs. (5), (3), (7), and (8) has several interesting and computationally important properties: 1) it is a concave function and the search region in dual space is a convex set defined by Eq. (6); 2) it is continuous and it has continuous first derivatives with respect to λ_q over the entire region defined by Eq. (6); 3) the first derivatives of $\ell(\lambda)$ are easily available because it can be shown (see p. 47 of Ref. 5) that they are given by primal constraints, that is

$$\frac{\partial \ell}{\partial \lambda_q}(\lambda) = u_q(\alpha) - \bar{u}_q = \sum_{b=1}^{B} C_{bq}\alpha_b - \bar{u}_q \tag{9}$$

4) since the second derivatives of $\ell(\lambda)$ are given by

$$\frac{\partial^2 \ell}{\partial \lambda_q \partial \lambda_k}(\lambda) = \frac{\partial u_q}{\partial \lambda_k}(\alpha) = \sum_{b=1}^{B} C_{bq} \frac{\partial \alpha_b}{\partial \lambda_k} \tag{10}$$

it follows from Eqs. (7) and (8) that discontinuities of the second derivatives exist on hyperplanes in the dual space defined by

$$\sum_{q\in Q_R} \lambda_q C_{bq} = \frac{w_b}{[\alpha_b^{(L)}]^2} \tag{11}$$

and

$$\sum_{q\in Q_R} \lambda_q C_{bq} = \frac{w_b}{[\alpha_b^{(U)}]^2} \tag{12}$$

Maximization Algorithm

In this section the computational algorithm used to solve the explicit dual problems [Eqs. (5), (3), (7) and (8)], corresponding to each of the approximate primal problems [Eqs. (1-4)], is described. This algorithm is a second-order Newton-type of maximizer (called DUAL 2) which exploits the special form of the explicit dual function $\ell(\lambda)$. It seeks the maximum of the dual function $\ell(\lambda)$ subject to nonnegativity constraints [see Eqs. (5) and (6)], in a sequence of subspaces with gradually increasing dimension, such that the dimensionality of the maximization problem rarely exceeds the number of strictly critical behavior constraints by more than one. The DUAL 2 algorithm has been found to be very efficient in practice, even though there are hyperplanes in the dual space where the second partial derivatives of $\ell(\lambda)$ are not unique [see Eqs. (11) and (12)].

The DUAL 2 algorithm involves iterative modification of the dual variable vector as follows:

$$\lambda_{t+1} = \lambda_t + d_t S_t \tag{13}$$

where S_t denotes the modification direction in dual space and d_t the distance of travel along that direction. Alternatively in scalar form the modification is given by

$$\lambda_{q,t+1} = \lambda_{qt} + d_t S_{qt}; \quad q \in Q_R \tag{14}$$

Let F_{qk} denote the elements of the matrix of second partial derivatives of $\ell(\lambda)$, then from Eq. (10)

$$F_{qk} = \frac{\partial^2 \ell}{\partial \lambda_q \partial \lambda_k}(\lambda) = \sum_{b=1}^{B} C_{bq} \frac{\partial \alpha_b}{\partial \lambda_k} \tag{15}$$

and from Eqs. (7) and (8) it follows that

$$\frac{\partial \alpha_b}{\partial \lambda_k} = \begin{cases} 0 & \text{if} \quad \alpha_b^{(L)} = \alpha_b \\ -\dfrac{\alpha_b^3 C_{bk}}{2 w_b} & \text{if} \quad \alpha_b^{(L)} < \alpha_b < \alpha_b^{(U)} \\ 0 & \text{if} \quad \alpha_b = \alpha_b^{(U)} \end{cases} \tag{16}$$

Substituting Eqs. (16) into Eq. (15) gives

$$F_{qk} = \frac{\partial^2 \ell}{\partial \lambda_q \partial \lambda_k}(\lambda) = -\frac{1}{2} \sum_{b \in \bar{B}} \frac{C_{bq} C_{bk}}{w_b} \alpha_b^3 \tag{17}$$

where the summation on the index b is over the set of free primal variables

$$\bar{B} = \{ b \mid \alpha_b^{(L)} < \alpha_b < \alpha_b^{(U)} \} \tag{18}$$

In the DUAL 2 algorithm the Newton method is used to seek the maximum of the dual function $\ell(\lambda)$ in various dual subspaces

$$M = \{ q \mid \lambda_{qt} > 0; \quad q \in Q_R \} \tag{19}$$

which exclude those λ_q components that are not currently positive. The move direction in such a dual subspace is given by

$$\underset{\sim}{S}_t = -[\underset{\sim}{F}(\lambda_t)]^{-1} \nabla \underset{\sim}{\ell}(\lambda_t) \tag{20}$$

where the subscript tilde (~) indicates that the collapsed vector (matrix) includes only those components (elements) corresponding to strict positive values of the dual variables at λ_t (i.e., entries for $\lambda_{qt} > 0$ only).

If the initial starting point in dual space is such that $[\underset{\sim}{F}(\lambda_t)]$ is nonsingular, and additional nonzero components $\lambda_q > 0$ are added one at a time, each subsequent $[\underset{\sim}{F}(\lambda_t)]$ will be nonsingular. When solving the dual problem corresponding to the first approximate primal problem, it is convenient to select the starting point so that the only nonzero dual variable corresponds to the most critical constraint (based on the structural analysis of the primal design used to generate the first approximate primal problem). For dual problems corresponding to subsequent approximate primal problems, the starting point is given by the dual variable values at the end of the previous dual function maximization. Use of this procedure in DUAL 2 further reduces the computational effort expended.

The DUAL 2 algorithm is outlined in the block diagram shown in Fig. 2. Given a set of values for the dual variables λ_{qt}; $q \in Q_R$ (see block 1) attention is directed to identifying the set of nonzero dual variables M (block 2). The integers in the set M define a dual subspace and in that subspace the maximum of the dual function $\ell(\lambda)$ is sought subject to nonnegativity constaints (see block 3). Let the maximum of $\ell(\lambda)$ in the subspace defined by the set M be denoted as λ_M. At λ_M evaluate the first partial derivatives of $\ell(\lambda)$ with respect to

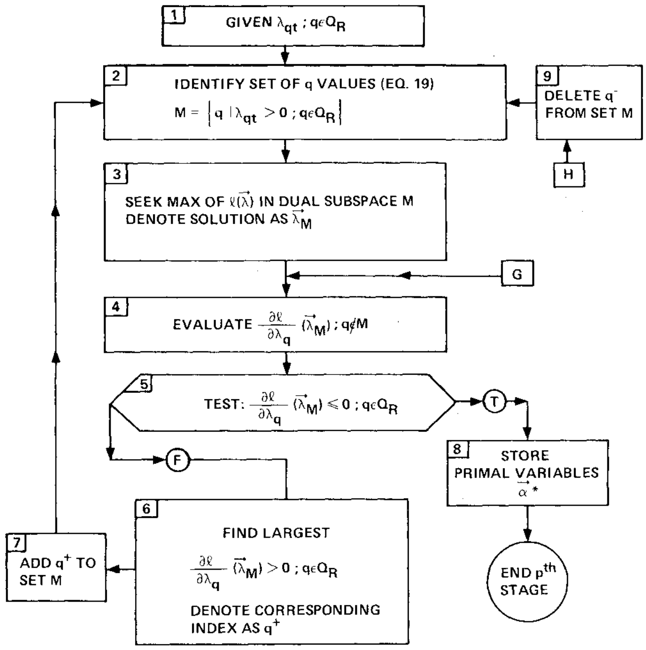

Fig. 2 DUAL 2 algorithm block diagram.

those λ_q not included in the subspace defined by the set M (block 4). Test to see if the maximum of $\ell(\lambda)$ in the dual space ($q \in Q_R$) has been obtained (block 5), if so store the primal variables corresponding to the current dual variables λ_M, end the stage and go to the overall design process convergence test (see Fig. 1). Otherwise, if any of the first partial derivatives $\partial \ell / \partial \lambda_q(\lambda)$; $q \in Q_R$ are positive, find the largest one, denote the corresponding index as q^+ (block 6), add this component to the set M [increasing the dimensionality of the dual space by one (block 7)], and continue to seek the maximum of the dual function $\ell(\lambda)$ corresponding to the current approximate primal problem.

The procedure used to find the maximum of $\ell(\lambda)$ in the dual subspace M (see block 3, Fig. 2) is explained in detail in Ref. 7. Here it suffices to know that either the maximum of $\ell(\lambda)$ is found in the subspace M [such that $|\nabla(\lambda_m)| \leq \epsilon$], in which case control returns to G in Fig. 2; or the search for the maximum of $\ell(\lambda)$ in the subspace M leads to a point where one of the current set of positive λ_q's becomes zero [before $\nabla \ell(\lambda_t) \leq \epsilon$] and it is necessary to delete that component from the subspace M (block 9, Fig. 2), in which case control returns to H in Fig. 2. The search for the maximum of $\ell(\lambda)$ in the dual subspace M involves iterative modification of the dual variable vector according to Eq. (13) and the move directions are given by Eq. (20). However, the procedure does not seek the maximum of the dual function along the direction $\underset{\sim}{S}_t$, rather it is designed to assure that either: 1) a regular Newton method unit step [$d_t = 1$ in Eq. (13)] is taken, without any change in the set of free‡ primal variables; or 2) the move distance is selected so that the value of the dual function increases. In either case provisions are included to insure that the step length d_t does not exceed the maximum step length d_{max} along $\underset{\sim}{S}_t$, where d_{max} is determined so that none of the $\lambda_{q,t+1}$ in the current set $q \in M$ becomes negative.

Scope of the ACCESS 3 Program

The basic ideas set forth in this paper, combining approximation concepts and dual methods, have been implemented in an approximation concepts code for efficient

‡A primal variable is said to be "free" if it has not taken on its upper or lower bound value $\alpha_b^{(U)}$ or $\alpha_b^{(L)}$ [see Eqs. (7) and (8)]. A change in the set of free primal variables signals that one or more second order discontinuity planes [see Eqs. (11) and (12)] have been crossed.

structural synthesis (ACCESS 3) computer program. This research type computer program was written by modifying the ACCESS 2 program,[4,8] essentially adding in the dual formulation as well as the DUAL 1 and DUAL 2 maximization algorithms. Herein, attention is restricted to the DUAL 2 algorithm for pure continuous variable problems. The DUAL 1 algorithm, which implements the extension of dual methods to mixed discrete-continuous design variables, will be presented in a sequel paper. The ACCESS 3 program contains all the ACCESS 2 capabilities as a subset and the data preparation formats are fully compatible.[9]

The ACCESS 3 program assumes that the structural topology, configuration, and material are preassigned parameters given by the user. The topology is specified via element-node connectivity data; the configuration is established by giving nodal positions (for the undeformed system) relative to a fixed reference coordinate system; and the given material is represented by its specific weight, stiffness, strength, and thermal expansion properties. The program treats sizing quantities (e.g., truss cross-sectional areas and thicknesses of shear panels or membrane elements) as design variables. The ACCESS 3 code accepts user supplied side constraints on the continuous design variables; and a rather general capability for design variable linking is also built into the program. Move limits can also be specified restricting the percentage change in the design variables within each stage of the overall iterative design process.

The program includes provisions for guarding against a variety of failure modes including strength, deflection, slope (relative deflections), and natural frequency limits. For truss members independent tension and compression allowables can be specified. In shear panels and isotropic membrane elements, where multiaxial stress states exist, strength constraints are introduced by limiting the value of the equivalent stress based on the distortion energy criterion. In the orthotropic membrane elements used to model fiber composite lamina at a preassigned orientation, three separate strength failure criteria options are available: the maximum strain criterion, stress interaction formulas, or the Tsai-Azzi criterion (see Refs. 4, 8, and 9). These strength failure criteria for fiber composite lamina take into account differences in the longitudinal, transverse, and shear allowables as well as differences in the tension and compression allowables.

The program also contains provisions for placing lower and upper limits on the first several natural frequencies. In addition to the structural mass, which varies as the sizing design variables change, fixed nodal masses can also be prescribed. For example, these fixed nodal masses can be used to simulate fuel inertia or engine masses in wing problems. Since attention is being restricted to the DUAL 2 algorithm the only frequency constraint approximation of interest herein is the first order Taylor series representation of $\lambda = \omega^2$ in terms of the independent reciprocal design variables after linking. It is noted in passing, that when the NEWSUMT algorithm for pure continuous design variables is employed,[7,10] there are three distinct approximation options available for frequency constraints.

The set of finite element types available in ACCESS 3 is the same as that in its precursor program ACCESS 2. They include bar (TRUSS), isotropic constant strain triangle (CSTIS), orthotropic constant strain triangle (CSTOR), isotropic symmetric shear panel (SSP), pure shear panel (PSP), and thermal shear panel (TSP) element types. A detailed description of the basic characteristics of the six element types currently included will be found in Appendix A of Ref. 8. All finite element types include provisions for representing thermal and body force loads. For each of several distinct loading conditions, temperature change and gravity field loads may be specified. These design variable dependent loads are included in addition to specified external applied loading conditions. The external applied loads may take the form of specified pressure loadings and/or given nodal forces for each loading condition. The objective function in ACCESS 3 is taken to be the total weight of the idealized finite element representation of the structural system.

The ACCESS 3 computer program is a research type program; however, it is capable of treating example problems that are large enough to clearly demonstrate the generality and efficiency that can be achieved by combining approximation concepts and dual methods. Research programs such as ACCESS 3 provide a knowledge and experience base on which to build full scale analysis/synthesis capabilities for widespread application in industry. The current problem size limits of ACCESS 3 are due primarily to the restriction that the compact vector form of the system stiffness and mass matrices must fit in core simultaneously. A further discussion of restrictions and limitations applicable to both ACCESS 2 and 3 will be found in Refs. 8 and 9.

Numerical Examples

A sample of numerical results obtained with ACCESS 3 for various continuous variable structural optimization problems is presented in this section. Special attention is focused on results obtained with the DUAL 2 algorithm and efficiency is assessed using comparable results obtained with the previously available[4] NEWSUMT algorithm. It should be recalled that ACCESS 2 (see Ref. 4) employed a SUMT type of optimization scheme based on the quadratic extended penalty function set forth in Ref. 11 and the unconstrained minimizations were carried out using the Newton method. The numerical results reported here indicate that the improved analysis/synthesis capability developed by combining dual methods and approximation concepts is remarkably efficient. Computational effort expended in the optimization portion of the program is reduced dramatically in representative examples (by at least a factor of 10) and the total computer times required to converge the overall optimization process is also reduced significantly. All examples will be presented in summary form; however, detailed input data can be found in Refs. 2, 4, and 5. Also a more extensive body of computational experience with ACCESS 3 is reported in Ref. 7.

Numerical results for several well studied standard truss problems with stress, deflection, and minimum size constraints have been obtained using both the DUAL 2 and the NEWSUMT optimizer options of the ACCESS 3 program (see Table 1). These examples were run as test cases to verify the program and also to provide a basis for assessing efficiency improvement. Without exception, the final weights, material distributions, and critical constraint sets obtained are identical for practical purposes. Since Ref. 2 contains complete tabular input data for the test problems, column 2 of Table 1 gives the problem number designation used in Ref. 2. Table 1 presents the final weights, number of analyses, CPU time in optimizer, and total CPU time for solving each of four truss design problems using the DUAL 2 and the NEWSUMT optimizer options. Examination of Table 1 reveals that using the DUAL 2 option drastically reduces the CPU time spent in the optimizer (by at least a factor of 10). It is also seen that the number of stages (analyses) required for convergence when using the DUAL 2 option is equal to or less than the number of analyses required for convergence when using the NEWSUMT option. The results in Table 1 also show that the total CPU time required to obtain an optimum design is substantially lower when the DUAL 2 algorithm is employed. As a matter of convenience similar results for the swept and delta wing examples, that are to be discussed subsequently, are also included in Table 1.

Attention is now directed to the idealized swept wing structure depicted in Fig. 3. The structure is taken to be symmetric with respect to the *x-y* plane which corresponds to the wing middle surface. The upper half of the swept wing is

Table 1 Summary of results for example problems

Problem name	Ref. 2 Problem no.	Final weight, kg (lb) NEWSUMT	DUAL 2	No. of analyses NEWSUMT	DUAL 2	CPU time, optimizer, s NEWSUMT	DUAL 2	CPU time, total, s NEWSUMT	DUAL 2
10 bar truss	3	2309 (5090)	2296 (5061)	13	13	1.62	0.14	5.88	4.39
25 bar truss	5	247.3 (545.2)	247.3 (545.2)	7	6	1.02	0.05	5.20	2.75
72 bar truss	6	176.3 (388.6)	172.2 (379.7)	10	5	1.84	0.07	12.86	5.86
63 bar truss	7B	2776 (6120)	2775 (6118)	14	13	100	8	146	53
Swept wing	9A	1123 (2475)	1118 (2464)	10	5	4.5	0.5	37	19
Delta wing	10[a]	6110 (13,470)	5810 (12,810)	29	15	145	2	719	261

[a] Finite element nodal geometry only (see Ref. 4 for other data).

Table 2 Iteration history data—swept wing example

Analysis No., n	Normalized weight W_n/W_I [a] NEWSUMT, 0.5×2	ACCESS 3 DUAL 2 Unscaled	Scaled	ACCESS 1, NEWSUMT[2]	Rizzi[12]
1	1.000	1.000	0.779	1.000	1.000
2	0.715	0.584[b]	0.584	0.682	0.579
3	0.607	0.498	0.498	0.545	0.541
4	0.549	0.497	0.497	0.510	0.517
5	0.522	0.497	0.497	0.500	0.501
6	0.509			0.498	0.496
7	0.504			0.497	0.497
8	0.501				0.497
9	0.500				0.497
10	0.499				0.497
...					...
17					0.496
		CPU time, s[c]			
Total	37.0	19.3		21.5	—
Analysis	30.8	17.7		17.0	—
Optimizer	4.5	0.5		3.1	0.44[d]

[a] W_I total initial weight = 2249 kg (4,959 lb). [b] Infeasible design. [c] IBM 360/91. [d] CDC 7600 (for comparison, time should be multiplied by 5).

Table 3 Material distribution—swept wing example

Linked design variable region		Thickness/minimum gage[a] ACCESS 3 NEWSUMT	DUAL 2	ACCESS 1, NEWSUMT[4]	Rizzi[12]
CST	1	10.26	10.21	10.20	10.17
(Skin)	2	8.935	8.890	8.885	8.865
	3	7.900	7.850	7.845	7.815
	4	6.550	6.470	6.480	6.465
	5	5.845	5.795	5.765	5.550
	6	5.175	5.160	5.135	4.771
	7	1.004	1.000	1.000	1.000
SSP	1	1.492	1.343	1.466	1.612
(Web)	2	1.108	1.107	1.089	1.000
	3	2.160	2.807	2.220	1.728
	4	1.818	1.785	1.766	2.282
	5	10.41	9.870	10.45	10.97
	6	1.411	1.523	1.866	4.765
	7	4.703	4.668	4.519	4.451
	8	4.052	3.935	4.000	2.980
	9	1.642	1.503	1.628	1.747
	10	2.211	2.524	2.456	3.018
	11	2.894	3.335	3.218	5.070
Skin weight/W_I [b]		0.4427	0.4400	0.4393	—
Web weight/W_I [b]		0.0565	0.0570	0.0573	—
Total weight/W_I [b]		0.4992	0.4970	0.4966	0.4964
No. of Analysis		10	5	8	17

[a] Minimum gage 0.0508 cm (0.020 in.). [b] W_I Total initial weight = 2249 kg (4,959 lb).

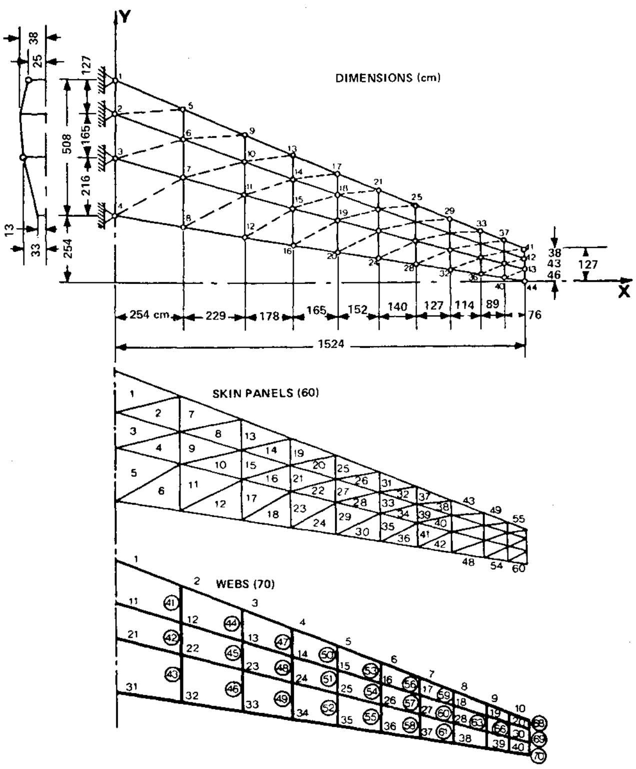

Fig. 3 Swept wing analysis model.

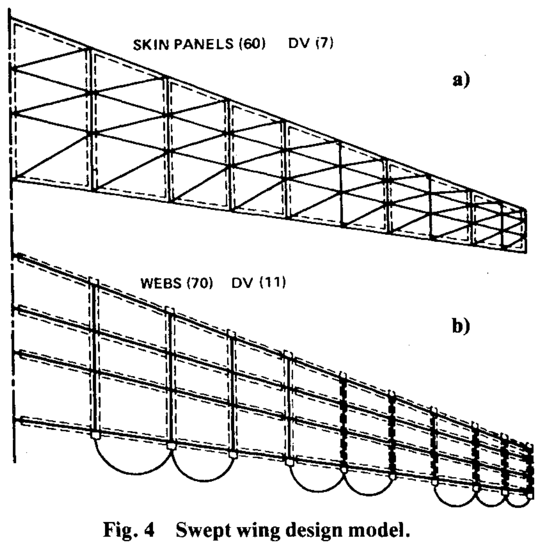

Fig. 4 Swept wing design model.

modeled using 60 constant strain triangular (CST) elements to represent the skin and 70 symmetric shear panel elements for the vertical webs. Extensive but plausible design variable linking is employed and the total number of independent design variables after linking is 18; 7 for the skin thicknesses (see Fig. 4a) and 11 for the vertical webs (see Fig. 4b). The wing is subject to two distinct loading conditions and the material properties are representative of a typical aluminum alloy. Detailed input data for this problem including nodal coordinates, design variable linking, applied nodal loadings, and material properties will be found in Ref. 2, Tables 51-57. The minimum weight optimum design of this idealized swept wing structure, subject to the following constraints is sought: 1) tip deflection is not to exceed 152.4 cm (60 in.) (at nodes 41 and 44 in Fig. 3); 2) Von Mises equivalent stress is not to exceed 172,375 kN/m^2 (25,000 lb/in.^{2}) in any finite element; and 3) minimum gage of skin and web material is not to be less than 0.0508 cm (0.020 in.).

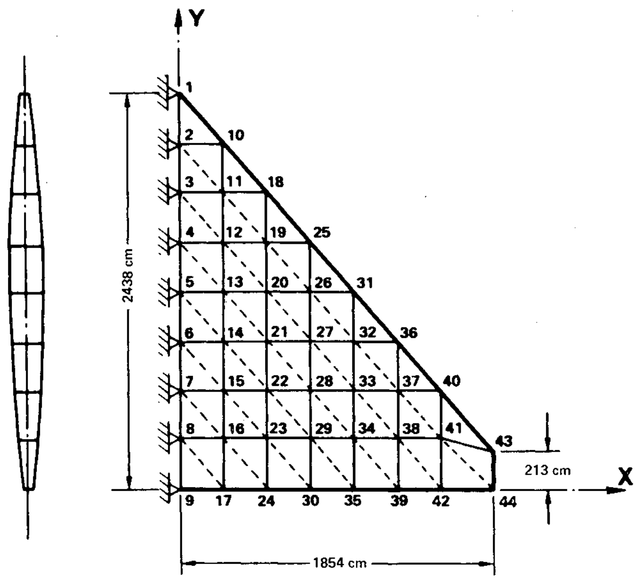

Fig. 5 Delta wing analysis model.

This problem was run using both the NEWSUMT and DUAL 2 optimizer options available in the ACCESS 3 program. Iteration history and runtime data for the final designs obtained are given in Table 2. In Table 2, the weight values W_n are normalized with respect to the initial weight [W_I = 2249 kg (4959 lb)] while in Table 3 the thicknesses describing the material distribution are normalized with respect to the minimum gage [i.e., 0.0508 cm (0.020 in.)]. Previously reported results from Refs. 2 and 12 are included in Tables 2 and 3 to facilitate comparison. In Table 2 the unscaled DUAL 2 results correspond to a sequence of "exact" solutions obtained for each approximate primal problem and the weight value marked with a superscript b does not correspond to a feasible design. The scaled DUAL 2 results in Table 2 are all feasible and critical. They are obtained by scaling the exact solutions for each approximate problem so that a feasible design with at least one strictly critical constraint is obtained.

By examining Tables 2 and 3 it is seen that the final weights and material distributions obtained by using the NEWSUMT and DUAL 2 options of ACCESS 3 are for practical purposes essentially the same. These results are also seen to be in excellent agreement with those previously reported in Refs. 2 and 12. Comparing the DUAL 2 results with the NEWSUMT results, both obtained with the ACCESS 3 program, it is seen that the advantages of using the dual approach are: 1) the number of structural analyses required for convergence drops from 10 (NEWSUMT) to 5 (DUAL 2); 2) the final weight obtained with DUAL 2 after 5 analyses is 0.5% lower than the final weight generated by the NEWSUMT option after 10 stages; 3) the total CPU time is reduced from 37.0 s for NEWSUMT to 19.3 s for DUAL 2; and 4) the computer times expended in the optimizer part of the ACCESS 3 program are 4.5 and 0.5 s for NEWSUMT and DUAL 2, respectively. It is important to point out that the computer time expended in the DUAL 2 optimizer remains small, despite the relatively large dimension of the dual space at each stage. Among the 268 behavior constraints, 40 are retained as potentially critical and 11 are found to be active by DUAL 2. This means that the operational dimensionality of the dual problem is 11, which is rather close to the dimensionality of the primal problem (18 independent design variables). Finally, it should be noted that the DUAL 2 final design has the following set of critical constraints: 1) minimum gage size for the skin elements 49-60 (see Fig. 3) in the outboard skin panel; 2) combined stress criteria in skin elements 8, 14 and 20 under load condition 1; and 3) combined stress criteria in web elements 20, 21, 30 and 58 under load condition 1 as well as web elements 3, 5, 20 and

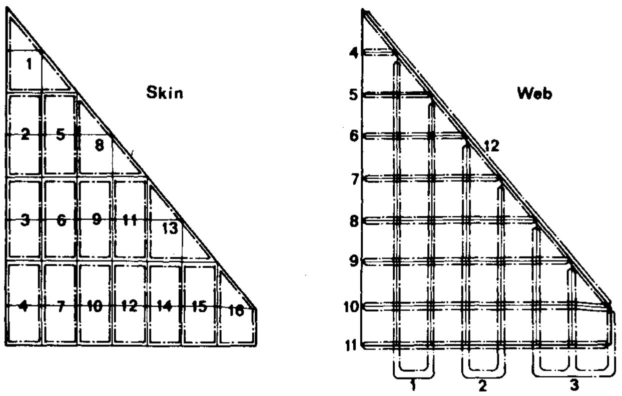

Fig. 6 Delta wing design model.

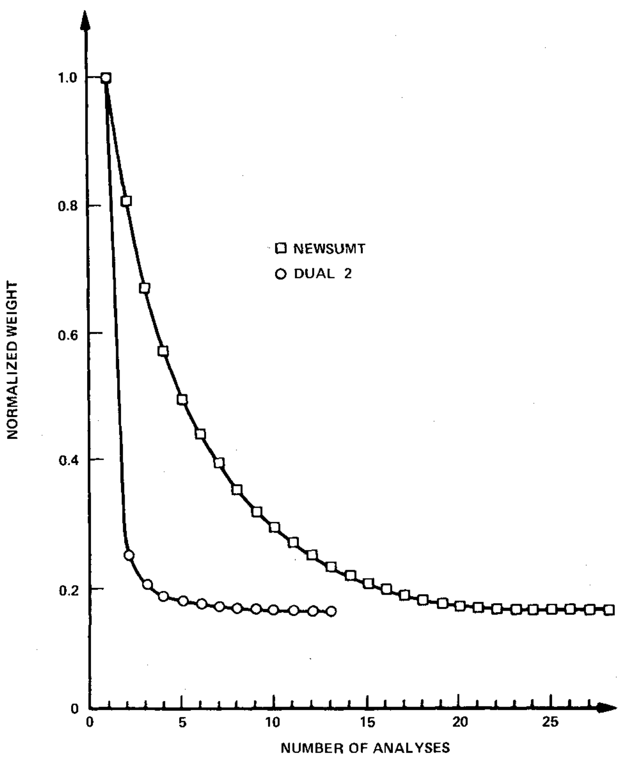

Fig. 7 Iteration histories for delta wing example.

42 under load condition 2. This set of critical constraints at the DUAL 2 final design is essentially the same as that reported for the final design found in Ref. 2 (see Fig. 25 of Ref. 2).

The last example treated here is a thin (3% thickness ratio) delta wing structure with graphite-epoxy skins and titanium webs. This problem has been previously studied in Refs. 2-4. The structure is symmetric with respect to its middle surface which corresponds to the x-y plane in Fig. 5. The skins are assumed to be made up of 0, ±45, and 90 deg high strength graphite epoxy laminates. It is understood that orientation angles are given with respect to the x reference coordinate in Fig. 5; that is, material oriented at 0 deg has fibers running spanwise while material oriented at 90 deg has fibers running chordwise. The laminates are required to be balanced and symmetric and they are represented by stacking 4 constant strain triangular orthotropic (CSTOR) elements in each triangular region shown in Fig. 5. Therefore, the skin is represented by $4 \times 63 = 252$ CSTOR elements while the webs are modeled using 70 symmetric shear panel (SSP) elements. According to the linking scheme depicted in Fig. 6, it can be seen that the total number of independent design variables is equal to 60 made up as follows: 16 for 0 deg material, 16 for

Table 4 Iteration history data for delta wing problem

Analysis No., n	NEWSUMT W_n/W_I [a]	NEWSUMT $\omega_n/\bar{\omega}$ [b]	DUAL 2 W_n/W_I	DUAL 2 $\omega_n/\bar{\omega}$
1	1.000	1.415	1.000	1.415
2	0.809	1.325	0.246	1.008
3	0.669	1.258	0.193	0.981
4	0.566	1.198	0.165	0.969
5	0.491	1.147	0.171	1.004
6	0.438	1.105	0.161	0.994
7	0.391	1.064	0.159	1.002
8	0.344	1.021	0.157	1.004
9	0.308	1.005	0.154	1.003
10	0.283	1.005	0.152	1.002
11	0.261	1.005	0.151	1.001
12	0.241	1.005	0.150	1.001
13	0.224	1.005	0.149	1.001
14	0.210	1.005	0.148	1.000
15	0.197	1.005	0.148	1.000
20	0.162	1.002		
25	0.156	1.001		
29	0.155	1.001		
		CPU time, s		
Total	719		261	
Analysis	564		252	
Optimizer	145		2	

[a] W_I total initial weight = 39,381 kg (86,820 lb). [b] $\bar{\omega}$ minimum required fundamental frequency = 2 Hz.

±45 deg material, 16 for 90 deg material, and 12 for the web material. The wing is subjected to a single static loading condition that is roughly equivalent to a uniformly distributed loading of 6.89 kN/m^2 (144 lb/ft^2). Static deflection constraints of ±254 cm (±100 in.) are imposed at the wing tip nodes. The strength requirements for the laminated skins are based on the maximum strain failure criterion.[4,8,9] In addition, the fundamental natural frequency is required to be larger than 2 Hz, while fixed masses simulate fuel in the wing.[4] Minimum gage requirements are also specified [0.0508 cm (0.02 in.) for the titanium webs and 0.0127 cm (0.005 in.) for the fiber composite lamina].

The problem examined here has its genesis in an interesting scenario presented in Ref. 4. Using an all titanium structure it was possible to obtain a satisfactory wing weight even when a 2-Hz lower limit was placed on the fundamental frequency. However, when fixed mass simulating fuel was added to the wing, it was necessary to introduce fiber composite skins in order to avoid an unacceptable fourfold increase in the minimum weight. Initially a high modulus graphite epoxy fiber composite was employed; however, subsequent consideration of temperature induced stresses made it necessary to switch to a high strength graphite epoxy material. In this paper the final version of the delta wing problem (case 3B of Ref. 4) will be reconsidered using the dual method approach. This example involves: 1) the use of laminated high strength graphite epoxy skins; 2) temperature change effects; 3) consideration of fixed fuel mass; and 4) a 2-Hz lower limit on the fundamental natural frequency (which is a primary design driver).

Numerical results for this delta wing example are given in Tables 4 (iteration histories) and 5 (final designs). These tables include results obtained with both the DUAL 2 and NEWSUMT options of the ACCESS 3 program. The corresponding iteration histories are depicted graphically in Fig. 7. Since the fundamental natural frequency constraint is the main design driver in this example, the ratio $\omega_n/\bar{\omega}$ (where $\bar{\omega} = 2$ Hz) as well as the normalized weight for each design in the sequence is given in Table 4. Note that designs 3, 4, and 6 in the DUAL 2 sequence are slightly infeasible with respect to the frequency constraint. Tables 4 and 5 as well as Fig. 7 show that the advantages of the dual method approach are significant for the delta wing example: 1) the number of

Table 5 Material distribution—delta wing example

Linked design Variable region	Fiber orientation, deg	Thickness/minimum gage Initial design	NEWSUMT	DUAL 2
1	0	30	1.92	1.56
	±45	30	1.52	1.00[a]
	90	30	1.68[a]	1.00[a]
2	0	120	5.62	9.38
	±45	100	3.26[a]	1.00[a]
	90	20	1.26[a]	1.00[a]
3	0	300	29.82	18.16
	±45	200	4.68[a]	2.56[a]
	90	60	1.00[a]	1.00[a]
4	0	300	186.9	232.0
	±45	200	7.08[a]	4.78[a]
	90	60	2.74	1.60
5	0	120	2.70	4.18
	±45	100	1.96	1.00
	90	20	1.00	1.00[a]
6	0	300	22.76	11.12
	±45	200	6.32	2.18
	90	60	1.00	1.00[a]
7	0	300	151.2	165.9
	±45	200	6.76	2.58
	90	60	3.18	2.36
8	0	40	1.38	1.00
	±45	40	1.00	1.00
	90	20	1.00	1.00
9	0	200	16.18	6.10
	±45	100	8.12	3.54
	90	40	1.00	1.00
10	0	200	116.0	125.1
	±45	100	8.10	5.02
	90	40	1.00	1.00
11	0	200	8.22	2.66
	±45	100	8.90	4.48
	90	40	1.00	1.00
12	0	200	74.94	79.78
	±45	100	11.16	7.72
	90	40	1.00	1.00
13	0	40	2.10	1.00
	±45	40	5.96	3.06
	90	20	1.00	1.00
14	0	60	40.56	42.64
	±45	20	11.70	9.32
	90	20	1.00	1.00
15	0	60	17.08	17.90
	±45	20	9.00	8.88
	90	20	1.00	1.00
16	0	20	5.70	5.26
	±45	20	2.06[a]	2.48[a]
	90	20	1.00[a]	1.00[a]
Skin wt./total initial wt		0.9587	0.1391	0.1334
Web wt./total initial wt.		0.0413	0.0161	0.0142
Total wt./total initial wt.		1.0	0.1552	0.1476
No. of analyses			29	15

Notes: 1) Minimum gage = 0.0127 cm (0.005 in.). 2) Total initial weight = 39,381 kg (86,820 lb). [a]Denotes transverse tension strain in bottom skin within 5% of limiting value.

structural analyses required for convergence falls from 29 (NEWSUMT) to 15 (DUAL 2); 2) the final weight obtained by DUAL 2 after 15 analyses is 5% lower than the final weight generated by NEWSUMT after 29 analyses; 3) the total computer time is reduced from 719 s for NEWSUMT to 261 s for DUAL 2; and 4) the computer times expended in the optimizer part of the program are 145 s and 2 s for NEWSUMT and DUAL 2, respectively. (These times are for runs on the IBM 360/91 computer at CCN, UCLA.)

Looking at the final designs generated by NEWSUMT and DUAL 2 (Table 5) it can be seen that the two designs are similar to each other. The smaller weight achieved with DUAL 2 appears to be due, at least in part, to the larger number of design variables that reach minimum gage. In both cases most of the fiber composite material in laminated skins is oriented spanwise with relatively small amounts placed at ±45 deg. Over most of the skin the 90 deg or chordwise material is minimum thickness critical (i.e., 0.0127 cm (0.005 in.)]. The web material distribution has been omitted in Table 5, in view of the small contribution that the web makes to the total weight of the wing (11%). The final designs generated by both NEWSUMT and DUAL 2 are governed primarily by the critical frequency constraint. However, several skin strength constraints are also critical in design variable regions 1-6 and 16 (see Fig. 6). These critical strength constraints are transverse tension strain limits in the bottom skin for material oriented at ±45 and 90 deg.

Conclusions

The fundamental reasons underlying the efficiency achieved by combining approximation concepts and dual methods are as follows.

1) Dual methods exploit the special algebraic structure of the approximate problem generated at each stage [see Eqs. (1-4)].

2) Since the approximate primal problem [Eqs. (1-4)] at each stage is convex, separable, and algebraically simple, it is possible to construct an explicit dual function [see Eqs. (5, 3, 7, and 8)].

3) Most of the computational effort is expended on finding the maximum of the dual function [Eq. (5)] subject only to simple nonnegativity constraints on the dual variables [Eq. (6)].

4) The dimensionality of each dual space, namely the number of critical and potentially critical inequality constraints retained during that stage (Q_R), is relatively small for many problems of practical interest.

5) The DUAL 2 algorithm has been especially devised so that it seeks the maximum of the dual function [Eq. (5)] subject to non-negativity constraints [Eq. (6)] by operating in a sequence of dual subspaces with gradually increasing dimension, such that the dimensionality of the maximization problem never exceeds the number of strictly critical constraints by more than one (see Maximization Algorithm section).

6) Finally by seeking the exact solution of each approximate problem [Eqs. (1-4)] using the DUAL 2 option, rather than the partial solution of each approximate problem using the NEWSUMT option, the number of stages needed to converge the overall iterative design process (see Fig. 1) is usually reduced.

The joining together of approximation concepts and dual methods provides further insight into the relationship between mathematical programming methods and optimality criteria techniques.[13] It is well known that the essential difficulties involved in applying conventional optimality criteria methods are those associated with identifying the correct critical constraint set and the proper corresponding subdivision of passive and active design variables (e.g., see Refs. 14-16). Special maximization algorithms, such as DUAL 2, designed to implement the dual method approach intrinsically deal with and resolve these two crucial difficulties. The subdivision of passive and active design variables is dealt with by the closed form relations [Eqs. (7) and (8)] expressing the primal design variables (α_b) as functions of the dual variables (λ_q). Identification of the critical constraint set ($\lambda_q > 0$) is automatically handled by taking the nonnegativity constraints on the dual variables [Eq. (6)] into account when seeking the maximum of the dual function [Eq. (5)]. Thus, the combining of approximation concepts and dual methods leads to a perspective where optimality criteria techniques are seen to reside within the general framework of a mathematical programming approach to structural optimization.

It is concluded, based on the results reported in this paper, that combining approximation concepts with dual methods provides a firm foundation for the development of rather general and highly efficient structural synthesis capabilities. Although ACCESS 3 is a research type program of limited scope, a substantial body of computational experience[7] supports the conclusion that combining approximation concepts and dual methods leads to a powerful and efficient capability for minimum weight optimum sizing of structural systems subject to stress, deflection, slope, minimum gage, and natural frequency constraints. Using these methods the computational effort expended in the optimization portion of the program has been reduced to a small fraction (e.g., less than 1% in the delta wing example with the DUAL 2 option) of the modest total run times required to obtain a minimum weight design.

Acknowledgments

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